The Pioneer anomaly in the context of the braneworld scenario

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Abstract

We examine the Pioneer anomaly - a reported anomalous acceleration affecting the Pioneer 10/11, Galileo and Ulysses spacecrafts - in the context of a braneworld scenario. We show that effects due to the radion field cannot account for the anomaly, but that a scalar field with an appropriate potential is able to explain the phenomena. Implications and features of our solution are analyzed.

1 Introduction

Studies of radiometric data from the Pioneer 10/11, Galileo and Ulysses have revealed the existence of an anomalous acceleration on all four spacecrafts, inbound to the Sun and with a (constant) magnitude of $a_A \simeq (8.5 \pm 1.3) \times 10^{-10} \ ms^{-2}$. Extensive attempts to explain this phenomena as a result of poor accounting of thermal and mechanical effects, as well as errors in the tracking algorithms used, have shown to be unsuccessful [1], despite a recent claim otherwise [2].

The two Pioneer spacecrafts follow approximate opposite hyperbolic trajectories away from the Solar System, while Galileo and Ulysses describe closed orbits. This, together with the fact that the three designs are geometrically distinct, explains the lack of an "engineering" solution for the anomaly. However, it prompts for a much more intriguing question: what is the fundamental, and possibly new, physics behind this anomaly?

To answer this, many proposals have been advanced. The range of ideas is quite diverse and we mention some of them: Yukawa-like or higher order corrections to the Newtonian potential [3]; a scalar-tensor extension to the standard gravitational model [4]; Newtonian gravity as a long wavelength excitation of a scalar condensate inducing electroweak symmetry breaking [5]; interaction of the spacecrafts with a long-range scalar field coupled to gravity [6, 7] ; an inverse time dependence for the gravitational constant G [8]; a length or momentum scale-dependent

cosmological term in the gravitational action functional [9]; a 5-dimensional cosmological model with a variable extra dimensional scale-factor and a static external space [10]; a local Solar System curvature for light-like geodesics arising from the cosmological expansion [11]; similarly, a recent work argues that the reported anomaly is related with the cosmological constant at the scale of the Solar System [12], even though this would lead to a repulsive force; an interaction with "mirror gas" or "mirror dust" in the Solar System [13]; a superstrong interaction of photons or massive bodies with the graviton background, yielding a constant acceleration, proportional to the Hubble constant [14]; expansion of solid materials on board deep space probes and contraction on Earth due to a curved stress field arising from repetitive tidal action [15]; an expanded PPN-framework so to incorporate a direct effect on local scales due to the cosmic space-time expansion [16]; a result of flavor oscillations of neutrinos in the Brans-Dicke theory of gravity [17]; a theory of conformal gravity with dynamical mass generation, including the Higgs scalar (capable of reproducing the standard gravitational dynamics and tests within the Solar System, and yet leaving room for a Pioneer-like anomaly on small bodies) [18]; a gravitational frequency shift of the radio signals proportional to the distance to the spacecrafts and the density of dust in the intermediate medium [19]; resistance of the spacecrafts antennae as they traverse interplanetary dust [20]; a gravitational acceleration $a \propto r^{-2}$ for a constant $a \gg a_0 = 10^{-10} \ ms^{-2}$ and $a \propto r^{-1}$ for $a \ll a_0$ [21]; clustering of dark matter in the form of a spherical halo of a degenerate gas of heavy neutrinos around the Sun [22] - amongst a few other suggestions put forward. It is interesting to mention that in higher-curvature theories of gravity where the gravitational coupling is asymptotically free and which have been much discussed in the context of the dark matter problem [23, 24, 25], a stronger gravitational coupling is expected on large scales and hence, at least in principle, to a Pioneer-like anomalous acceleration.

In this work we consider the Pioneer anomaly in the context of the braneworld scenario. We use the Randall-Sundrum model and variations to show that gravitational effects such as the one due to the radion field cannot explain the anomaly. We argue that a scalar field with a suitable potential implies that geodesics in this theory exhibit an extra constant attractive acceleration.

2   Braneworld Theories

A quite new range of possible scenarios arise in the context of braneworld theories. In these, one assumes our Universe to be a 3-dimensional world-sheet embedded in a higher dimensional bulk space. Considerations on the symmetries of the brane and its topological properties are then taken to constrain the evolution of matter on the brane and gravity on the brane and in

the bulk.

Braneworld theories are a fast developing trend in cosmology [26, 27, 28, 29, 30], whose main feature is to allow for a solution for the hierarchy problem, whether the typical mass scale of the bulk is comparable with the electroweak breaking scale, $M_{EW} \sim TeV$.

In this work, we shall consider the Randall-Sundrum braneworld model and some variations [26, 27]. One admits a scenario with two 3-branes embedded in (4+1)-dimensional space: a positive tension brane situated at $z = 0$ and a negative tension one at $z = z_c$. The *Ansatz* for the metric takes the form

$$ds^2 = e^{-2kz} g_{\mu\nu} dx^\mu dx^\nu + dz^2 \quad , \tag{1}$$

which is a solution of the 5-dimensional Einstein's equations and preserves Poincaré invariance on each brane. The constant k is a fundamental quantity of the theory and typically takes values of order $k \sim M_{Pl}$, the 4-dimensional Planck mass, which is dynamically generated from the bulk space Planck mass M_5,

$$M_{Pl}^2 = \frac{M_5^3}{k} \left(1 - e^{-2kz_c} \right) \quad . \tag{2}$$

This relation is obtained from the derivation of a 4-dimensional effective action by integrating the fifth dimension away. Notice that M_{Pl} depends weakly on the second membrane position, in the large kz_c limit. Physical masses on the positive brane, however, scale with this distance through the relationship $m = e^{-kz_c} m_0$ (a solution for the hierarchy problem is obtained for $kz_c \approx 15$). In traditional compactification schemes, the "warp" factor e^{-2kz} in the metric is absent, and hence integration over the fifth dimension yields only the volume of the bulk space, V_n; for a n-dimensional compact space one obtains $M_{Pl}^2 = M^{n+2} V_n$.

3 Braneworld Scenarios for the Pioneer anomaly

As a first attempt to explain the Pioneer anomaly within the context of braneworld theories, one could resort to the appearance of a tower of Kaluza-Klein massive tensorial perturbations to the metric. Three problems arise: all gravitons are ordered according to their mass, so that one cannot freely specify the range of one of them without affecting the whole tower. Most braneworld models consider one first light mode with cosmological range, and all ensuing modes to have sub-millimeter range. It is difficult to introduce an intermediate scale without abdicating from one of the two desired extreme cases.

The second problem refers to the fact that any Yukawa gravitational potential would affect all bodies within range, independently of their mass (as expected from the Equivalence Princi-

ple). This is in direct contradiction with the lack of an observed "anomalous" acceleration for the planets in the Solar System.

Thirdly, it has been shown that an Yukawa potential fitted to the observed effect would reveal the presence of a graviton with range $L \sim 200$ AU *and* a negative coupling $\alpha \sim -10^{-3}$ [3]. One can adjust the exact values, since they belong to a solution curve $L = L(\alpha)$ (the one presented is a rather "natural" choice, since $|\alpha| \ll 1$ and $L \sim 100$ AU). However, α is always negative. Since the coupling of modes is proportional to the normalization factor of their wave functions in the Kaluza-Klein reduction scheme, this would imply in a graviton with negative norm - which is unattainable within current braneworld theories, leading to instabilities at a quantum level. This latter issue shows that quantized tensor excitations are not the cause of the Pioneer anomaly.

If one considers instead a single membrane, then the periodic boundary condition disappears and the modes are no longer quantized. In the standard Randall-Sundrum scenario [27], this amounts to a modification of gravity at large distances, with the gravitational potential presenting a $1/r^2$ behaviour, typical of the five-dimensional propagation of gravity in the bulk. This behaviour also appears in models where the modes are quantized and dense, and their mass spectrum approaches a continuous distribution [30].

A more elaborate model was suggested in Ref. [29], which shows a similar behaviour of the gravitational potential. At large distances it also goes as $1/r^2$, while at intermediate distances it contains a repulsive logarithmic term. In both cases (and more exotic ones, see Refs. [30, 31] and references therein), a continuous spectrum of Kaluza-Klein excitations can be shown to be unable of explaining the Pioneer effect.

An alternative solution could be that the Pioneer anomaly reflects the influence of the radion field $f(x)$, a scalar perturbation of the metric corresponding to translational zero modes, related to relative motion of the two branes. Since its moduli is usually stabilized due to some *ad hoc* potential whose effect is superimposed on the usual warped metric (see Ref. [32] and references therein), one could view the radion as an additional field on each brane. We discuss its effect on a test particle and verify to which extent it could provide an explanation for the Pioneer anomaly.

Following Ref. [33], the induced metric on the positive ($z = 0$) brane which includes this perturbation is, in Gaussian coordinates ($g_{zz} = -1$ and $g_{z_\mu} = 0$),

$$h_{\mu\nu}^+ = \eta_{\mu\nu}[1 - 2kf(x)] + \frac{1}{2k}f_{,\mu\nu} \quad , \tag{3}$$

where $f(x)$ is constrained by the requirement that in the vacuum

$$\Box^2 f = 0 \quad , \tag{4}$$

and

$$\Box^2 \equiv \left(\eta^{\lambda\sigma} f_{;\lambda}\right)_{;\sigma} = \eta^{\lambda\sigma} f_{,\lambda\sigma} - \eta^{\lambda\sigma} \Gamma^{\alpha}_{\lambda\sigma} f_{,\alpha} \tag{5}$$

is the 4-dimensional d'Alembertian. This arises from the Israel junction conditions imposed on each brane so to ensure the $\mathcal{Z}_{\in}$ symmetry. The metric $\eta^{\lambda\sigma}$ is the unperturbed one on the positive brane. In the presence of matter, Eq. (4) is not homogeneous and is related to the trace of the energy-momentum tensor of the matter distribution.

We now search for a solution $f(r)$ that is both static and spherically symmetric. We work in spherical coordinates where f' denotes $f_{,r}$, and $f_{,\theta} = f_{,\phi} = f_{,t} = 0$. In the weak field limit, one has $\eta_{00} = 1 + 2U$ and $\eta_{rr} = -(1 - 2U)$, where $U = -GM_{\odot}/r \equiv -C/r$ is the Newtonian gravitational potential; in units where $c = \hbar = 1$, $C = 1.5 \ km$. Thus, one obtains

$$\Box^2 f = -f'' - \frac{2}{r} f' = 0 \quad , \tag{6}$$

neglecting the term due to the curvature, which is proportional to $U'(r)f'(r)$.

A simple solution is given by $f(r) = k^{-1}(A/r + B)$, with $B \ll 1$ a dimensionless constant, $[A] = L$ and $A/r \ll 1$ within the desired range. Hence, the induced metric becomes

$$h^+_{00} = [1 + 2U(r)][1 - 2kf(r)]$$
$$h^+_{rr} = -[1 - 2U][1 - 2kf(r)] + k^{-1}f''(r) \quad . \tag{7}$$

We next proceed by considering the spacecrafts as point particles. Thus, they follow time-like geodesics of the obtained vacuum metric. The acceleration is then

$$\begin{aligned}
a^r &= -\Gamma^r_{00} = \frac{1}{2} h^{rr} \partial_r h_{00} = -\frac{1}{2}[1 + 2U] \left[\frac{1}{1 - 2kf - k^{-1}f''(r)} \right] \partial_r \left[(1 + 2U)(1 - 2kf)\right] \\
&\simeq -[1 + 2U][1 + 2kf + k^{-1}f''] \partial_r \left[U - kf - 2kUf\right] \\
&= -\frac{C + A}{r^2} + \frac{-2A^2 + 4AC + 2C^2}{r^3} + \frac{4A(3A - C)C}{r^4} \\
&\quad - \frac{2A(A + C + 8AC^2 k^2)}{k^2 r^5} + \frac{4AC(3A + C)}{k^2 r^6} - \frac{16A^2 C^2}{k^2 r^7} \quad ,
\end{aligned} \tag{8}$$

and one can see that no constant term arises, rendering this approach unsuitable to account for the discussed anomalous acceleration.

Another possible explanation for the Pioneer anomaly could be related with a bimetric theory exhibiting Lorentz symmetry breaking. A proposal along these lines was discussed in Ref. [34] and is briefly presented in the Appendix A.

In principle, a bimetric theory could arise from induced effects on the 3-brane of gravity in the bulk. However, as shown in Ref. [35], Goldstone modes resulting from the spontaneous breaking of coordinate diffeomorphisms carry no extra degrees of freedom due to the positions of the branes in curved spacetime, but rather manifest themselves in the form of an extra field in the energy-momentum tensor, involving the induced metric $\gamma_{mn} = \partial_m x^\mu \partial_n x^\nu g_{\mu\nu}(x)$. Thus, although invariance under general coordinate transformations does not hold along the fifth dimension, it is a symmetry on each brane. Since the resulting singular energy-momentum tensor cannot be treated as a "mass" term for the graviton (at least at the linear level), massive gravitons do not arise due to the interaction of the brane with gravity in the bulk. It is argued that this is crucial to preserve the r^{-2} long-range behaviour of gravity.

Nevertheless, it should be pointed out that long-range modifications to Newton's law have been the focus of many braneworld related proposals aiming to explain the accelerated expansion of the Universe (see e.g. Ref. [28] and references therein). It is important to realize that the above arguments are valid in the absence of topological obstructions on the brane. Hence, no extra degrees of freedom arise when the metric is taken as a dynamic variable under local, "small", diffeomorphisms. However, under large gauge transformations [36], where the gauge parameters assume different values in non-connected asymptotic regions and have different global behaviour, solitonic-type solutions are admitted.

These solutions are deformations, $h_{\mu\nu}$, of a particular metric solution $g^{(1)}_{\mu\nu}$ of the Einstein's equations with a source term given by the extra field arising from the breaking of diffeomorphism invariance along the fifth direction, such that the deformed metric $g^{(1)}_{\mu\nu} + h_{\mu\nu}$ is a solution of the same equation. We have verified that known solitonic-type solutions do not give rise to a constant acceleration term [37, 38] that could explain the Pioneer anomaly.

In what follows, we shall derive the general behaviour of a scalar field with a potential and calculate its effect on the motion of a test particle.

4   Scalar Field Coupled to Gravity

As a possible explanation for the Pioneer anomaly, we consider the effect induced by the presence of a scalar field ϕ with dynamics driven by a potential $V(\phi) \propto -\phi^{-\alpha}(r)$, with $\alpha > 0$. The form of this potential closely resembles that of some supergravity inspired quintessence models [39, 40, 41]. In the context of the braneworld scenario, the quintessence potential has

the form $V \propto \phi^{-\alpha}(t)$, with $2 < \alpha < 6$ [42]. Notice that, in this proposal, there is a spatial and not a time dependence. Also, the sign of our potential is reversed, so to yield a static *attractive* acceleration.

As usual, we assume a small perturbation to the Minkowsky metric and solve it in terms of the energy-momentum tensor of the field.

The metric can be written as

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \tag{9}$$

where in the weak field limit (in spherical coordinates)

$$(\eta)_{\mu\nu} = diag(1 + 2U(r), -1 + 2U(r), -r^2, -r^2 sin^2\theta) \quad , \tag{10}$$

and

$$h_{\mu\nu} = diag(f(r), -h(r), -h(r)r^2, -h(r)r^2 sin^2\theta) \quad . \tag{11}$$

Notice that the bimetric character arises from the assumption that the field ϕ expresses the effect of the induced metric arising from the spontaneously broken diffeomorphisms in the curved spacetime.

The Lagrangian density of the static scalar field takes the form

$$\mathcal{L}_\phi = \frac{1}{2}\eta_{\mu\nu}\partial_\mu\phi\partial^\nu\phi - V(\phi) = \frac{1}{2}\eta_{rr}(\phi')^2 + A^2\phi^{-\alpha} \quad , \tag{12}$$

where A is a constant. The scalar field obeys the equation of motion

$$\Box^2\phi + \frac{dV(\phi)}{d\phi} = 0 \quad , \tag{13}$$

which yields, neglecting a term proportional to $U'(r)\phi'(r)$

$$\phi''(r) + \frac{2}{r}\phi'(r) = \alpha A^2\phi^{-\alpha-1} \quad , \tag{14}$$

and admits the solution

$$\phi(r) = \left((2 + \alpha)\sqrt{\frac{\alpha}{8 + 2\alpha}}Ar\right)^{\frac{2}{2+\alpha}} \equiv \beta^{-1}r^{\frac{2}{2+\alpha}} \quad . \tag{15}$$

Notice that this solution is singular at $r = 0$. Its regularization is discussed in Appendix B. Thus, in terms of r, the potential and gradient terms are given by

$$V(\phi(r)) = -A^2\beta^\alpha r^{-\frac{2\alpha}{2+\alpha}} \quad , \tag{16}$$

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and

$$\frac{1}{2}\left(\phi'(r)\right)^2 = A^2\left(\frac{\alpha}{4+\alpha}\right)\beta^\alpha r^{-\frac{2\alpha}{2+\alpha}} = -\left(\frac{\alpha}{4+\alpha}\right)V(\phi(r)) \ . \tag{17}$$

The Lagrangian density is given in the Newtonian limit by

$$\mathcal{L}_\phi = -\frac{4}{4+\alpha}V(\phi(r)) = \frac{4A^2}{4+\alpha}\beta^\alpha r^{-\frac{2\alpha}{2+\alpha}} \ . \tag{18}$$

The energy-momentum tensor of the scalar field is obtained by the expression

$$T_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi - \eta_{\mu\nu}\mathcal{L}_\phi \ , \tag{19}$$

so that its components are given by

$$
\begin{aligned}
T_{00} &= -\eta_{00}\mathcal{L}_\phi = -[1+2U(r)]\frac{4A^2}{4+\alpha}\beta^\alpha r^{-\frac{2\alpha}{2+\alpha}} \\
T_{rr} &= \phi'(r)^2 - \eta_{rr}\mathcal{L}_\phi = (2+\alpha)\frac{2A^2}{4+\alpha}\beta^\alpha r^{-\frac{2\alpha}{2+\alpha}} \\
T_{\theta\theta} &= -\eta_{\theta\theta}\mathcal{L}_\phi = r^2\frac{4A^2}{4+\alpha}\beta^\alpha r^{-\frac{2\alpha}{2+\alpha}} \\
T_{\varphi\varphi} &= -\eta_{\varphi\varphi}\mathcal{L}_\phi = r^2 sin^2\theta\frac{4A^2}{4+\alpha}\beta^\alpha r^{-\frac{2\alpha}{2+\alpha}} \ ,
\end{aligned}
\tag{20}
$$

where we have assumed that the spatial perturbation to the metric is very small, $h(r) \ll 1$. The trace is given by

$$T \equiv T^\alpha_\alpha = \eta^{\mu\nu}T_{\mu\nu} = -(8+\alpha)\frac{2A^2}{4+\alpha}\beta^\alpha r^{-\frac{2\alpha}{2+\alpha}} \ . \tag{21}$$

We now turn to the linearized Einstein's equations:

$$\frac{1}{2}\nabla^2 h_{\mu\nu} = \kappa\left(T_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}T\right) \ , \tag{22}$$

where $\kappa = 8\pi G$. The $0-0$ component is

$$f''(r) + \frac{2}{r}f'(r) = 2\kappa\left(1 - \frac{2C}{r}\right)A^2\beta^\alpha r^{-\frac{2\alpha}{2+\alpha}} \tag{23}$$

whose solution is, for $\alpha \neq 2$,

$$f(r) = (2+\alpha)^2 A^2\kappa\beta^\alpha r^{-\frac{2\alpha}{2+\alpha}}\left(\frac{Cr}{\alpha-2} + \frac{r^2}{12+2\alpha}\right) \ , \tag{24}$$

and, for $\alpha = 2$,

$$f(r) = \sqrt{\frac{3}{2} \frac{A\kappa r}{2}} - \sqrt{6} AC\kappa \, log\left(\frac{r}{1 \, m}\right) \quad . \tag{25}$$

Notice that we have dropped the homogeneous solution of Eq. (23) of the form $1/r$ since it can be absorbed by the Newtonian term. Therefore, we obtain for the acceleration

$$a_r = -\frac{C}{r^2} + (2+\alpha)A^2\kappa\beta^a r^{-\frac{2\alpha}{2+\alpha}}\left(\frac{C}{2} - \frac{r}{6+\alpha}\right) \quad . \tag{26}$$

For $\alpha = 2$, it reads

$$a_r = -\frac{C}{r^2} - \sqrt{\frac{3}{2} \frac{A\kappa}{4}} + \sqrt{\frac{3}{2} \frac{AC\kappa}{r}} \quad . \tag{27}$$

Eqs. (26) and (27) indicate that a constant anomalous acceleration exists only for $\alpha = 2$. The first term is the Newtonian contribution, and one can identify the constant term with the anomalous acceleration $a_A = 8.5 \times 10^{-10} \, ms^{-2}$, setting

$$A = 4\sqrt{\frac{2}{3} \frac{a_A}{\kappa}} = 4.7 \times 10^{42} \, m^{-3} \quad . \tag{28}$$

The remaining term, proportional to r^{-1}, is much smaller than the anomalous acceleration for $4C/r \ll 1$, that is, for $r \gg 6 \, km$; it is also much smaller than the Newtonian acceleration for $r \ll 2.9 \times 10^{22} \, km \approx 100 \, Mpc$, clearly covering the desired range. It can be shown that higher order terms affect the third term in Eq. (27) and yield a negligible correction to the first term.

For consistency, we now show that $h(r) \ll 1$. For that, we use the $r - r$ component of Eq. (22) with $\alpha = 2$:

$$h''(r) + \frac{2}{r}h'(r) = -\frac{A\kappa}{\sqrt{6}r} \quad , \tag{29}$$

whose solution is $h(r) = -A\kappa r/2\sqrt{6}$. This term is negligible for $r \ll 5000 \, Mpc$, confirming the validity of the approximation. One can improve the previous result of Eq. (27) to first order in $h(r)$:

$$a^{(1)} = \frac{a_A}{1 + h(r)} \simeq a_A(1 - h(r)) = -\frac{C}{r^2} + \frac{5}{6}\sqrt{\frac{3}{2} \frac{AC\kappa}{r}} - \sqrt{\frac{3}{2} \frac{A\kappa}{4}}\left(1 - \sqrt{\frac{2}{3}}AC\kappa\right) - \left(\frac{A\kappa}{4}\right)^2 r \,, \tag{30}$$

which, asides from the small correction to a_A and a $5/6$ factor in the r^{-1} term, introduces a linear term which is negligible for $r \ll 5 \times 10^{29} \, Mpc$.

Thus, we see that an anomalous acceleration is a clear prediction of our model within the Solar System. Hence, an hypothetical dedicated probe to confirm the Pioneer anomaly [43, 44]

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does not need to venture into too deep space to detect such an anomalous acceleration, but just to a distance where it is measurable against the regular acceleration and the solar radiative pressure, actually approximately from Saturn onwards.

It can be shown that the potential $V(\phi) \propto -\phi^{-2}$ is the only one that can account for the anomalous acceleration. Indeed, from Eq. (23) we can write in the same approximation

$$\frac{1}{r^2}[r^2 f'(r)]' = \chi \kappa V(r) \eta_{00}(r) \quad , \tag{31}$$

where χ is a dimensionless factor depending on the potential ($\chi = -2$ for $\alpha = 2$). Hence,

$$f'(r) = \frac{\chi \kappa}{r^2} \int r^2 V(r) \eta_{00}(r) \quad . \tag{32}$$

Since, from Eq. (27), the anomalous acceleration is given by

$$a_r = -\frac{1}{2} f'(r) = -\frac{\chi \kappa}{2r^2} \int V(r) r^2 \eta_{00}(r) \quad , \tag{33}$$

and $\eta_{00} = 1 - 2C/r$, one concludes that the potential must have the form $V(r) = V(\phi(r)) = -A/r$ or $V(r) = B$, where B is a constant. The constant potential solution also provides an anomalous acceleration, but yields a conflicting linear term:

$$f_{(const)} = -2BC\kappa r + B\kappa r^2/3 \tag{34}$$

and hence

$$a_{r\ (const)} = -\frac{C}{r^2} - BC\kappa + \frac{B\kappa r}{3} \quad . \tag{35}$$

By identifying the constant term with the anomalous acceleration, one immediately finds $B = 0.03\ kg\ m^{-3}$; as a result, the linear term dominates the constant acceleration for $r \gg 1.5\ km$. Hence, we consider the case $\chi = -2$ or $\alpha = 2$.

Indeed, the equation of motion (13) can be written as

$$\nabla^2 \phi = \frac{1}{r^2}[r^2 \phi'(r)]' = \frac{V'(r)}{\phi'(r)} \quad , \tag{36}$$

and therefore

$$\phi'(r)[r^2 \phi'(r)]' = A \quad . \tag{37}$$

Thus, the only real solution of this differential equation is given, up to a constant factor, by $\sqrt{r}$. Hence, $r \propto \phi^2(r)$ and $V(\phi) \propto -\phi^{-2}(r)$, as argued.

With $\alpha = 2$, the energy of this scalar field grows logarithmically. This can be seen as a manifestation of a global symmetry breaking, as in the case of global cosmic strings, where the same behaviour is found [46].

Notice that these results apply only to the case of bodies with negligible mass, such as the considered spacecrafts. In the case of planets, endowed in general with rotation and spin, their dynamics are described by the solution of Eq. (22) with the appropriate energy-momentum tensor $T_{\mu\nu} = T_{\mu\nu}^{\phi} + T_{\mu\nu}^{planet}$, with $T_{\mu\nu}^{\phi} \ll T_{\mu\nu}^{planet}$. Hence, no anomalous acceleration is expected for celestial objects.

Furthermore, in what concerns the nature of the source of the scalar field, we assume it is the Sun and that matter couples with the scalar field in the following way:

$$\mathcal{L}_{int} = \phi \sum_i f_i \bar{\psi}_i \psi_i \quad , \tag{38}$$

where, ψ_i stands for different fermion species and f_i are couplings constants.

We now look at a possible breaking of the Equivalence Principle. It can be shown (see [47, 48] and references therein) that changes to geodesic motion arise (to first order) from spatial variations of mass, proportionally to $m'(r)/m(r)$. Hence, one must look at changes induced on the mass of a test particle. These will occur if the scalar field acquires a vacuum expectation value, such as in the standard case of the Higgs mechanism, or in the cosmologically relevant cases ([47]).

In our proposal, the potential for the scalar field is negative and monotonically increasing. Hence, the effective potential $V_{eff}(\phi) = V(\phi) + \phi \sum_i f_i n_i$, where n_i is the density of different matter species coupled to the scalar field, does not develop a minimum. Therefore, no mass changes as well as no violations of the Equivalence Principle are expected.

Finally, we look at the propagation time delay induced by this bimetric theory. If it is not negligible, then the value of the anomalous acceleration should be corrected, since the radiometric data which supports it is based on the Doppler effect [1].

For simplicity, we consider only the worst-case scenario: a test particle travelling at a constant velocity of $v = 10^{-5}c$ (approximately the current velocity of Pioneer 10/11), following a linear path away from the Sun and at a distance r_0 from it. If the light signal is emitted with a proper period T_0, its relation with the period recorded on Earth, T, is

$$\begin{aligned}
T &= \int_{T_0} \sqrt{[1 + f(r(t))] - v^2} \, dt \simeq \int_{T_0} \left[\sqrt{1 - v^2} + \frac{f(r)}{2\sqrt{1 - v^2}} \right] \frac{dt}{v} \\
&= \int_{\Delta r} \left[\sqrt{1 - v^2} + \sqrt{\frac{3}{2}} A\kappa \frac{\left[r/4 - C \, log\left(\frac{r}{1\,m}\right) \right]}{\sqrt{1 - v^2}} \right] \frac{dr}{v} \quad , \tag{39}
\end{aligned}$$

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where $\Delta r = vT_0$. This yields

$$T = \left(\sqrt{1 - v^2} + \sqrt{\frac{3}{2}} \frac{A\kappa r}{4\sqrt{1 - v^2}} \right) T_0 \ , \tag{40}$$

where we have used that $r \gg 1.5 \ km$ and $r \gg \Delta r$. Hence, the bimetric effects on the time delay can be safely disregarded for $r \ll 3000 \ Mpc$.

5 Conclusions and Outlook

In this paper we have investigated the possibility of explaining the Pioneer anomaly within the framework of braneworld scenarios. We have eliminated both "new" tensorial (massive gravitons) as well as scalar (radion) degrees of freedom as candidates for a solution. We found that the anomalous acceleration could be due to the presence of a negative potential scalar field, with a potential $V \propto -\phi^{-2}(r)$ similar to some supergravity inspired quintessence models.

Notice that the approach considered here, contrary to naive thinking, has no implications for the puzzle of the rotation curve of the galaxy. Indeed, assuming a galaxy to be virialized, one can describe the rotation of a layer at a distance r of the galactic core as $v^2(r) = GM(r)/r$, where $M(r)$ is the total mass inside the layer. Observation shows that $v^2(r)$ displays a steady rise until a threshold of about 10 kpc ($\sim 2 \times 10^9 AU$), and a constant plateau from there (see eg. Ref. [45]). This leads one to model $M_{Gal}(r) \sim r$ and hence to postulate the presence of dark matter.

Starting from Eq. (27), we can derive a different expression for $v^2(r)$:

$$v^2(r) = \frac{C^*}{r} - \sqrt{\frac{3}{2}} C^* \kappa + \sqrt{\frac{3}{2}} \frac{A^* \kappa r}{4} \ , \tag{41}$$

where the superscript * refers to galactic values. This curve does not describe the observed data and would lead one to model $M(r)$ as

$$M(r) = M_{Gal}(r) \left[1 + \sqrt{\frac{3}{2}} A^* \left(\kappa r + \frac{\kappa r^2}{4C^*} \right) \right] \ . \tag{42}$$

This clearly differs from usual dark matter models due to the higher order terms in Eq. (42). However, likewise the case of planets, the test masses here cannot be viewed as point-like objects, and hence have to be treated with its energy-momentum tensor. Therefore, we are lead to conclude that the origin of the flattening of the rotation curves of galaxies does not have its roots in the induced bimetric theory.

6 Appendices

6.1 Appendix A.

A bimetric theory of gravity was first proposed by Rosen [49]. Its action is given by

$$S = \frac{1}{64\pi G} \int d^4x \sqrt{-\eta} [\ \eta^{\mu\nu} g^{\alpha\beta} g^{\gamma\delta} (g_{\alpha\gamma|\mu} g_{\beta\delta|\nu} - \frac{1}{2} g_{\alpha\beta|\mu} g_{\gamma\delta|\nu}) + \mathcal{L}_M(g_{\mu\nu})]\ , \qquad (43)$$

where the vertical line | denotes covariant derivation with respect to the background metric $\eta_{\mu\nu}$ only, and $\mathcal{L}_M$ is the matter Lagrangian density. The resulting equation for the dynamical gravitational field is given by:

$$\Box_\eta g_{\mu\nu} - g^{\alpha\beta} \eta^{\gamma\delta} g_{\mu\alpha|\gamma} g_{\nu\beta|\gamma} = -2\kappa (g/\eta)^{1/2} (T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T)\ , \qquad (44)$$

where $T \equiv T_{\mu\nu} g^{\mu\nu}$ and $\Box_\eta$ is the d'Alembertian operator with respect to $\eta_{\mu\nu}$. Notice that the momentum-energy tensor couples only to the dynamical metric $g_{\mu\nu}$. We can always choose coordinates in which $(\eta_{\mu\nu}) = diag(-1, 1, 1, 1)$, the Minkowsky metric, and $(g_{\mu\nu}) = diag(-c_0, c_1, c_1, c_1)$, where c_0 and c_1 are parameters that may vary on a Hubble H^{-1} timescale [49].

This theory explicitly breaks Lorentz invariance. This is better understood by resorting to the Parametrized Post-Newtonian (PPN) formalism: a systematic expansion of first-order $1/c^2$ terms in the Newtonian gravitational potential and related quantities [50]. It turns out that any metric theory of gravitation can be classified according to ten PPN parameters: $\gamma, \beta, \xi, \alpha_1, \alpha_2, \alpha_3, \zeta_1, \zeta_2, \zeta_3, \zeta_4$. These are the linear coefficients of each possible first-order term (generated from rest mass, energy, pressure and velocity), and relate a particular theory with fundamental aspects of physics: conservation of linear and angular momentum, preferred-frame and preferred-location effects, nonlinearity and space-curvature per unit mass, etc.

Einstein's General Relativity, the most successful theory up to date, exhibits a set of PPN parameters with $\beta = \gamma = 1$, the remaining being equal to zero. Rosen's bimetric theory has $\beta = \gamma = 1$, but a non-vanishing $\alpha_2 = c_0/c_1 - 1$ coefficient. This indicates that the theory is semi-conservative (it lacks angular momentum conservation) and exhibits preferred-frame effects: Lorentz invariance is broken and the Strong Equivalence Principle does not hold. It is worth pointing out that the breaking of Lorentz invariance has been recently very much discussed. Indeed, possible signatures of the breaking of this symmetry arise from ultra-high energy cosmic rays with energies beyond the Greisen-Zatsepin-Kuzmin cut-off, $E_{GZK} \simeq 4 \times 10^{19}\ eV$, (see Ref. [51] for a discussion on the astrophysical aspects of the problem), from the observation of gamma radiation from faraway sources with energies beyond $20\ TeV$, and from the longitudinal evolution of air showers created by ultra-high energy hadronic particles and which seem to imply

that pions are more stable than expected (for an update see [34, 52] and references therein). Of course, Lorentz invariance holds with great accuracy as observed deviations are quite small, $\delta < 3 \times 10^{-22}$ [53] from direct measurements, and even smaller from the study of ultra-high energy cosmic rays $\delta \simeq 1.7 \times 10^{-25}$ [54, 55].

By linearizing Eq. (44) in the vacuum, one obtains the wave equations for weak gravitational waves, whose solution is a wave propagating with speed $c_g = \sqrt{c_1/c_0}$. Thus, α_2 measures the relative difference in speed (measured by an observer at rest in the Universe rest frame) between electromagnetic and gravitational waves, inducing a time delay in the propagation of light signals [50]. This effect has been claimed to be put under test in a recent experiment using the close celestial alignment of Jupiter and the quasar $J0842 + 1835$. This analysis yield $c_g/c = 1.06 \pm 0.21$ [56], corresponding to $\alpha_2 = -0.11 \pm 0.35$. Note, however, that this result is still controversial, and an alternative interpretation suggests that it sets instead a limit on γ and α_1, actually $\gamma = 1$ and $\alpha_1 = 0$, being unrelated to the velocity of propagation of gravity [57].

A rigorous study of deviations between the Sun's spin axis and the ecliptic has led to the experimental constraint $|\alpha_2| < 1.2 \times 10^{-7}$ [58]. Improving this bound as well as finding new means of verifying its implications are clearly of key importance. Interestingly improvements on the measurement of Sun's oblateness (and ensuing spin) as well as on the PPN parameters β, γ and the combination $\eta \equiv 2 - \beta + 2\gamma$ are on the list of objectives of the ambitious BepiColombo mission to Mercury [59].

Due to their small mass, self-gravitation is also absent for the considered spacecrafts. Thus, these can be regarded as particles, which enables us to calculate their acceleration by simply computing the time-like geodesics of the full metric, $h_{\mu\nu} = \eta_{\mu\nu} + g_{\mu\nu}$.

In the weak field limit, $v \ll c$, one has

$$a^i \simeq -\Gamma^i_{00} = \frac{1}{2} h^{i\lambda} \partial_\lambda h_{00} \quad , \tag{45}$$

from which one can identify the radial anomalous component of the acceleration:

$$\vec{a}_A = c_1 \vec{\nabla}_r U - \frac{(1 - c_1)}{2} \vec{\nabla}_r c_0 \quad . \tag{46}$$

It can be immediately seen that, if c_0 and c_1 are homogeneous in space, the derived anomalous acceleration is not constant, which contradicts the observation. Therefore, we assume these parameters depend on the distance to the Sun, that is, $c_0 = c_0(r)$, $c_1 = c_1(r)$. Given the constraint

$$\alpha_2 = \left| \frac{c_0(r)}{c_1(r)} - 1 \right| < 4 \times 10^{-7} \quad , \tag{47}$$

we assume that both coefficients have the same r-dependence, so that α_2 is homogeneous. We consider the choice

$$c_0 = D \, r \quad , \quad c_1 = F \, r \quad , \tag{48}$$

with $D, F > 0$ so that the resulting anomalous acceleration is inbound. According to Ref. [58], $|D/F - 1| < 4 \times 10^{-7}$ and hence $D \simeq F$. Note that D cannot be exactly equal to F, since this implies that $\alpha_2 = 0$.

Substituting *Ansatz* Eq. (48) into Eq. (46), we find

$$a_A = -\frac{D}{2} \left[1 - \frac{F}{D} \frac{2C}{r} - Fr \right] \simeq -\frac{D}{2} \left[1 - \frac{2C}{r} - Dr \right] \quad . \tag{49}$$

Hence, $D = 2a_A = 1.9 \times 10^{-26} \ m^{-1}$ and we see that the distance-dependent contributions to a_A are negligible for r lying in the interval

$$[2C, D^{-1}] = [3 \ km, 3.5 \times 10^{14} \ AU] \quad , \tag{50}$$

which is consistent with the fundamental assumption that $c_0, c_1 \ll 1$.

Unfortunately, this elegant solution for the Pioneer anomaly cannot be taken seriously, given the behaviour of Rosen's theory in what concerns gravitational waves. Indeed, in Ref. [60], it is argued that Rosen's bimetric theory is fundamentally flawed, since it predicts dipole gravitational radiation beyond the limits measured from the pulsar $PSR \ 1913 + 16$ [61]. Moreover, the solution proposed by Rosen of considering a combination of retarded and advanced gravitational waves [62] implies in contradiction with the observed quadrupole gravitational radiation from binary pulsars [61]. Thus, Rosen's bimetric theory cannot be considered a viable theory of gravity.

6.2   Appendix B.

Equation (14) is not satisfied at the origin, the center of the Sun, as solution of Eq.(15) is singular at $r = 0$. However, this solution can be regularized in the standard way by introducing a source term in the Lagrangian density (12). This can be performed by considering a thin shell model, dividing the space into two regions, $r \geq r_0$ and $r < r_0$, with r_0 a positive constant. Assuming that the Pioneer anomaly is verifiable everywhere in the Solar System, it follows that r_0 is smaller than $R_\odot$, the Sun's radius - this assumption can be relaxed, since it is currently

not accessible to experiment, due to the suppression of the effect by the larger solar wind and Newtonian terms.

Naturally, one assumes that the solution of Eq.(15), $\phi(r) \equiv \phi(r)_+$, is valid for $r \geq r_0$. For $r < r_0$, we consider the solution

$$\phi(r) = \phi_-(r) \equiv C + \frac{\alpha}{6} A^2 C^{-\alpha-1} r^2 + \mathcal{O}\left(r^4\right) \quad, \tag{51}$$

where C is a regularization constant. Continuity of the scalar field at $r = r_0$ implies that C is a solution of $\phi_-(r_0) = \phi_+(r_0)$. Even though the full solution is now regular at $r = 0$, the derivative changes value at $r = r_0$ as $\phi'_+(r_0) \neq \phi'_-(r_0)$ and, as before, Eq.(14) is not satisfied at $r = r_0$. A suitable solution requires adding to Eq. (14) the term $\left(\phi'_+(r_0) - \phi'_-(r_0)\right) \delta(r - r_0)$, which demands an additional term in the Lagrangian density Eq. (12):

$$\mathcal{L}_\phi \rightarrow \mathcal{L}_\phi + \phi\left(\phi'_+(r_0) - \phi'_-(t_0)\right) \delta(r - r_0) \quad. \tag{52}$$

Hence, one can remove the "inner" solution $\phi_-(r)$ by taking the $r_0 \to 0$ limit and keeping the source term in Eq.(52).

Acknowledgments

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